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**Joseph Zeira**

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# Inequality and Mobility\*

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January 24, 2002

## Abstract

We construct a model where wage inequality, intergenerational mobility and the distribution of skills are all jointly determined in general equilibrium, which is solved analytically. We show that it is important to treat these variables jointly, as they have a significant effect on each other. The model is used to understand both empirical regularities and policy implications. The main result of the paper is that changes in education, including public education, lead to a negative correlation between inequality and upward mobility, while changes in productivity tend to lead to a positive correlation between inequality and upward mobility. Hence, the observations that some European countries tend to be more equal but less socially mobile than the US cannot be attributed to differences in public support to education, but rather to differences in productivity. Another result of the paper is that while public support to education benefits children of poor parents, it benefits children of skilled and rich parents by even more, improving their relative chances of success. Since parents care about their children, general public support to education can then increase the difference in welfare between skilled and non-skilled individuals, notwithstanding a reduction in wage inequality.

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# 1 Introduction

In recent years, there has been a growing interest among macroeconomists in issues of inequality, skill distribution and intergenerational mobility. This paper offers an integration of these economic issues in a unified theoretical framework. Our theory enables us to examine the relationships between these variables, their reaction to exogenous changes, including policy changes, and their implications for individual welfare levels.

The main motivation of this paper is to understand why there exists such substantial differences between countries with respect to these variables. Why are some countries more equal than others, and why are some countries more socially mobile? For example, Checchi, Ichino and Rustichini (1999) have recently shown that Italy is more equal but less mobile than the U.S. Bjorklund, Eriksson, Jantti, Raaum and Osterbacka (2001) show that in addition to being more equal, the Nordic countries are more mobile than the U.S. Dahan and Gavira (1999) report on large differences in mobility between Latin American countries. We believe that a general equilibrium model, where inequality and mobility are jointly and endogenously determined, can shed light on these stylized facts and on their causes.

Our motivation is not only empirical, but also based on theoretical considerations. Inequality, mobility, and distribution are strongly related and hence, need to be analyzed simultaneously. For example, mobility rates affect the distribution of skills. If wages are endogenous, this affects wages and wage inequality. But inequality also affects mobility, both as an incentive to upward mobility, and as a constraint on the poor to invest in skill acquisition. Note that a general equilibrium framework that studies inequality and mobility jointly is important also for welfare analysis. While people care about their own income, we should acknowledge that they also care about their children. Hence, mobility affects individual welfare, as it determines the chances of the individual's children improving their

social lot. This will prove to be important when analyzing the effects of various policies on inequality, mobility and distribution, as it tells us how these policies affect individual welfare levels. This might also be relevant for the issue of political support for such policies. Therefore, we believe that a key contribution of this paper is to provide a tractable model, where generations are linked through an altruistic motive and inequality, mobility and skill-distribution are determined endogenously.

Let us now describe the main elements of our model. Workers have two levels of human capital: they are either skilled or non-skilled. This leads to clear definitions of inequality, skill distribution and mobility in our model. Wage inequality is defined as the ratio between wages of skilled and non-skilled workers. Skill distribution is the ratio between the shares of skilled and non-skilled workers. For intergenerational mobility, we use two measures: upward mobility and inequality of opportunity. The former is the probability that children of non-skilled parents become skilled. The latter is the difference between the probabilities of becoming skilled for children of skilled and non-skilled parents, respectively.

Skill acquisition is costly. We assume that children cannot borrow against future human capital, implying that their parents must finance this cost. Parents differ in income and hence, in investment in their children's education, but we also assume that the productivity of this investment may depend on the parents' education. In other words, skilled parents have higher income but they are also more effective in providing education for their children. They know better what books to buy, what type of tutoring fits the needs of their children, what additional courses they should take, etc., which gives them an additional edge in supplying education for their children. Finally, education costs also depend on child inherent educational aptitude (innate ability), which is stochastic and assumed to be independent between generations.

As the main case, we assume individual aptitude to be known to both parent and child when the educational investment decision is undertaken. The parent allocates income between education of her child and her own consumption. Thus, there is a threshold level of aptitude above which children get education, which depends on parental income and education. The different thresholds for skilled and non-skilled parents determine the rates of mobility in the economy. In addition to the full information case, we also explore other informational assumptions, such as no information on the child's aptitude before starting school. This case is interesting as it creates a conflict of interests between parent and child. While the child wants her parent to pay as much as possible for education, to maximize her chances of becoming skilled, the parent also cares about her own consumption. We solve this conflict by using asymmetric Nash bargaining. Due to the conflict between parents and children, educational decisions in this case also depends on the relative social bargaining power of parents versus children.

As mentioned above, investment in education depends on income and hence, on inequality. We find inequality to have two opposite effects on upward mobility. On the one hand, future inequality creates an incentive to become skilled, and has a positive effect on upward mobility. We call this the *incentive effect*. On the other hand, higher inequality reduces the ability of non-skilled parents to pay for their childrens' education, since education is usually provided by skilled workers and its costs are indexed to their wage. This reduces upward mobility, and we call it the *distance effect*. We find that the incentive effect dominates if inequality is low, while the distance effect dominates if the economy is very unequal.

After endogenizing education decisions and thus, the distribution of skills, the model endogenizes wages as a function of the skill distribution, closing the general equilibrium in the economy. First, we use our model to examine the effect of a skill-biased technical

change, which raises the productivity of skilled workers by more than that of non-skilled. We show that it raises inequality, while its effect on upward mobility is ambiguous. If inequality is low, mobility rises, as the incentive effect dominates; if inequality is high, however, the distance effect dominates and mobility falls. We also examine the effect of such a change on the distribution of skills and find that the ratio of skilled to unskilled typically rises. In a very unequal economy, however, the distance effect might be strong enough to imply that the ratio of skilled to unskilled declines.

The paper examines the effect of various social and educational changes; a general improvement in the educational system, a reduction in the barriers to education faced by non-skilled parents and an increase in the bargaining power of parents. A common result for all these changes is a negative correlation between inequality and upward mobility; if a change reduces inequality, it raises upward mobility and *vice versa*. Returning to the empirical findings described above, we can say that while the difference between the US and the Nordic countries fits differences in the educational systems, the differences between the US and Italy is better attributed to productivity differences.

We also examine the effect on inequality and mobility of public support to education. Here, we reach two results. The first is that public support to education reduces inequality but raises upward mobility. The second is that public support to education may actually increase inequality of opportunity, namely that children of skilled parents gain more from public education than children of unskilled. Hence, the effect on social mobility of public support to education is mixed. While children of the poor do better in absolute terms, they might do worse relative to children of the rich.

Our paper belongs to a recently growing literature, which embeds the issues of inequality, distribution and mobility in a general equilibrium macroeconomic framework. This

literature has been spurred both by recent events in the world economy, namely the widening of wage gaps, and recent developments in economic research. Theoretical and empirical research has shown that income distribution affects the overall performance of the economy. Examples of this are Loury (1981), Galor and Zeira (1993), Banerjee and Newman (1993) and Aghion and Bolton (1997) for the role of inequality and distribution, and Durlauf (1996), Owen and Weil (1998), Maoz and Moav (1999), and Hassler and Mora (2000) for the role of intergenerational mobility. Examples of papers that have used cross country data to examine the effect of inequality on economic growth are Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996), and Barro (2000).

There is also a growing empirical literature on intergenerational mobility, strongly related to our paper. In addition to the papers cited above, we should mention Cooper, Durlauf and Johnson (1993) and the survey by Solon (1999). A recent empirical paper by Rubinstein and Tsiddon (1998) provides evidence that education is not only affected by parents' income but also by parents' education. Another relevant paper is Dynarski (2000). It shows that recently enacted U.S. programs that provide non-means-tested tuition subsidies, have increased the differences in college attendance rates between students from high- and low-income families, i.e., increasing inequality of opportunity, using our terminology. This conforms with our model results.

Here, we should again mention Checchi, Ichino and Rustichini (1999) who, in addition to the empirical comparison between Italy and the US, present a model of inequality and mobility to account for these results. Our model differs significantly from theirs, both in structure and results. We show that the difference between Italy and the US cannot be attributed to the public education system, as they claim. Furthermore, our model has a wider goal as it offers a general model to study the joint determination of inequality and



mobility and how these differ across countries.

The paper is organized as follows. Section 2 presents the basic model of inequality and mobility. Section 3 analyzes equilibrium and steady state with exogenous and endogenous wages, respectively. Section 4 analyzes the effects of a skill biased technical change. Section 5 analyzes the effects of various social and educational changes. Section 6 discusses the effect of public support to education and Section 7 the effect of redistributive policy. Section 8 analyzes the effect of social norms and Section 9 concludes.

## 2 The Model

Let us now describe our basic model. We assume that individuals live for two periods, consuming and inelastically supplying one unit of labor only in the second period of their life. In the first period, the individual chooses whether to become skilled (type  $s$ ) or non-skilled (type  $n$ ). Skilled workers earn more than unskilled workers, namely  $w_s > w_n$ , where  $w_j$  is the wage of type  $j \in \{s, n\}$  expressed in terms of output. To become skilled, agents need to learn, which is costly. To finance learning, a child needs the support of her parents. The reliance on parents is motivated by difficulties in borrowing against future human capital but also by our belief that parents play a crucial role in the learning process that cannot be perfectly substituted for by capital markets.

Individuals are born with different aptitudes for learning. We measure this aptitude by the index  $e$ , which is assumed to be drawn from a rectangular distribution over the unit interval and distributed *i.i.d.* across generations. To become skilled, a cost that falls in  $e$  must be paid. In other words, the lower the aptitude of the child, the more the parents must pay in order for their child to succeed at school and become skilled. We think of this cost as representing direct schooling costs like tuition, but also more indirect costs such as

tutoring and providing a good home environment. We assume education is produced using skilled labor, implying that the cost of education is proportional to the skilled wage. For  $j \in \{s, n\}$  denoting parental education, we assume that a child with aptitude  $e$  can become skilled if the amount spent on education  $z$  satisfies

$$z \geq \frac{1-e}{d_j} w_s, \quad (1)$$

where  $d_j$  represents the parental educational productivity and  $\frac{1-e}{d_j}$  should be interpreted as the required teacher time.<sup>1</sup> In line with the empirical evidence, such as in Rubinstein and Tsiddon (1998), we allow the educational productivity to vary with parental education level. We assume that  $d_s \geq d_n > 0$ , capturing the fact that skilled people (because they are educated) have more knowledge about how to efficiently educate a child. Associated with the decision on how *much* to spend on the childrens education are decisions on *how* to allocate these spending. For example, educating a child requires allocation of resources for buying books, but also which books to buy.<sup>2</sup>

We assume away capital markets, so children cannot borrow against future income and parental consumption is therefore equal to income net of taxes and transfers minus

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<sup>1</sup>It is straightforward to allow non-constant returns to scale in education by setting  $1-e \leq \left(\frac{\alpha w_j}{w_s}\right)^\gamma d_j$  with  $\gamma$  smaller or larger than unity.

<sup>2</sup>The differences in educational productivities can be modeled in an explicit way. Assume that the productivity of resources spent on education is (proportional to)  $e^{-\frac{1}{2}(x-a)^2}$  where  $x$  is the best action and  $a$  is the actual action. The best action  $x$  is unknown but known to everyone to be normally distributed with mean  $\mu$  and a variance  $\sigma$ . The expected return on educational expenditures is then  $E\left(e^{-\frac{1}{2}(x-\mu)^2}\right) = (1+\sigma^2)^{-1/2} \equiv d_n$ .

Suppose also that skilled parents receive a signal on the realization of  $x$ , normally distributed, unbiased and with precision  $P$ . Then, for skilled parents  $E\left(e^{-\frac{1}{2}(x-\mu)^2}\right) = \left(\frac{1+\sigma^2(1+P)}{1+P\sigma^2}\right)^{-1/2} \equiv d_s > d_n$ . Finally, assuming, realistically, that education entails a large number of such decisions and using the law-of-large numbers, implies that the return on educational expenditures becomes non-stochastic and larger for skilled parents.

educational investments.

Each individual receives one child in the second period of life and parents are altruistically linked to their children in a standard dynastic chain. Assuming logarithmic utility, the objective function of an old individual is

$$\ln(c) + \beta EU, \tag{2}$$

where  $EU$  is the expected utility of her child and  $\beta \in [0, 1)$  is the intergenerational discount factor. Without loss of generality, we assume that the individual discount factor between two periods of life is unity.

Assuming zero population growth, we normalize the size of each generation to unity and denote the amount of unskilled individuals in period  $t$  by  $N_t$ . Hence, the amount of skilled individuals is  $1 - N_t$ .

We next present the production side of the economy, where we consider two cases; exogenous and endogenous wages. In the first case, productivity and wages of both types of workers are fixed and do not depend on the amount of workers of each type. In this case  $w_s$  and  $w_n$  are fixed and do not change over time. In the second case, we assume that there is a competitive labor market where wages are given by marginal productivities.

Regarding our informational assumptions, we assume as the main case that the educational investment decision is taken after the child's aptitude has been revealed. We also analyze the case of no information about individual aptitudes when the educational investment decision is taken. In the appendix, we consider an intermediate case, where parent and child receive an imperfect signal on individual aptitude before the educational decision is made.

### 3 Equilibrium

This section describes the equilibrium of the model without government. The analysis of subsidies to education and redistributive taxation is deferred to Sections 6 and 7. We first deal with the case of exogenous wages and then turn to endogenous wages.

#### 3.1 Exogenous wages

We begin the analysis by studying the decision of investment in education. We assume that the aptitude is fully revealed to both parent and child at first period of life. As explained above, a parent of type  $j$  with a child with aptitude  $e$  has to pay an amount  $(1 - e)/d_j$  for her child to become skilled. However, parents will only be willing to pay this amount if their welfare, including the altruistic concern of their children, does not decrease if their child becomes skilled, i.e., if

$$\ln(w_j - \iota) + \beta V_s \geq \ln w_j + \beta V_n, \quad (3)$$

where  $V_j$  denotes the expected utility of an individual of skill type  $j$ , before knowing the aptitude of her offspring, namely in the first period of life. In order to find the maximum share of income, which a parent will pay to finance education for her child, we set the RHS equal to the LHS and get

$$\beta(V_s - V_n) = -\ln(1 - \alpha) \equiv E(\alpha), \quad (4)$$

where  $\alpha$  is the maximum share of income spent on education. Clearly, (4) defines a unique investment ratio  $\alpha \in [0, 1)$ , which is an increasing function of the expected utility difference, but independent of wage and educational productivity and, hence, of parental type. Given the type independent choice of  $\alpha$ , children with parents of type  $j$  with an aptitude  $e \geq 1 - \alpha d_j w_j \equiv e_j$  will become skilled. In contrast to  $\alpha$ , the threshold aptitude  $e_j$  depends on

type. The share of children with skilled parents who themselves become skilled is

$$1 - e_s = \alpha d_s, \quad (5)$$

while the share of children of unskilled parents who move upwards and become skilled is

$$1 - e_n = \alpha \frac{d_n}{I}, \quad (6)$$

where  $I \equiv w_s/w_n$  denotes wage inequality. Note that  $1 - e_n$ , which measures upward mobility in the economy, increases in  $\alpha$  and decreases in  $I$ .

A variable that captures another aspect of mobility is inequality of opportunity, which we define as

$$e_n - e_s = \alpha \left( d_s - \frac{d_n}{I} \right). \quad (7)$$

Inequality of opportunity measures the difference due to background between children of skilled and unskilled parents. It reflects both different levels of income and different productivities of education. We should note that if at least some individuals become skilled ( $\alpha > 0$ ), inequality of opportunity is positive whenever  $d_s > d_n$  and/or  $I > 1$ .

Now, we can calculate the expected utility<sup>3</sup>

$$V_j = -(1 - e_j) \left( \frac{\alpha + \ln(1 - \alpha)}{\alpha} \right) + \ln w_j + \beta V_n \quad (8)$$

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<sup>3</sup>To get this, note that

$$\begin{aligned} V_j &= \int_{e_j}^1 \ln(1 - \frac{(1-e)w_s}{d_j w_j}) de + \ln w_j + \beta V_n + \beta(1 - e_j)(V_s - V_n) \\ &= \frac{w_j d_j}{w_s} \int_0^\alpha \ln(1 - x) dx + \ln w_j + \beta V_n + \beta(1 - e_j)(V_s - V_n) \\ &= \frac{w_j d_j}{w_s} ((\alpha - 1) \ln(1 - \alpha) - \alpha) + \ln w_j + \beta V_n - (1 - e_j) \end{aligned}$$

implying that

$$\begin{aligned}\beta(V_s - V_n) &= \beta \left( \ln I - (e_n - e_s) \frac{\ln(1 - \alpha) + \alpha}{\alpha} \right) \\ &= \beta \left( \ln I - \left( d_s - \frac{d_n}{I} \right) (\ln(1 - \alpha) + \alpha) \right) \equiv \Delta V(\alpha)\end{aligned}\tag{9}$$

with  $\frac{\partial \Delta V(\alpha)}{\partial \alpha} = \beta \left( d_s - \frac{d_n}{I} \right) \frac{\alpha}{1 - \alpha} > 0$ . Note that the welfare difference between skilled and non-skilled does not only depend on wage inequality, but also on inequality of opportunity. Since parents care about their children, a larger inequality of opportunity leads to larger welfare differences. As we see, welfare differences are affected by the difference  $(e_n - e_s)$ , rather than, for example, the ratio of the thresholds. Therefore it appears natural to use the difference as the metric of inequality of opportunity.<sup>4,5</sup>

Equation (9) defines the welfare difference between skilled and non-skilled as a function of investment shares and wage inequality. Together, (4) and (9) define a unique equilibrium investment share and a welfare difference as a function of wage inequality. This equilibrium is depicted in Figure 1, where the steeper curve, starting at 0, represents (4) and the curve with an intercept is (9). The intercept of the latter equals  $\beta \ln I$ , implying that in equilibrium,  $\alpha \geq 0$ , and is equal to zero only if  $I = 1$ .

The maximum share of income invested in education,  $\alpha$ , is determined by the intersec-

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<sup>4</sup>It should also be noted that when  $d_s > d_n$ ,  $e_n - e_s = (1 - e_s) \left( 1 - \frac{d_n}{d_s} \right) > 0$ , is efficient in the sense that costs of education for the marginal individual from skilled and non-skilled homes, respectively, are equalized.

<sup>5</sup>In reality, other variables than parental education, income and innate ability may, of course, affect individual outcomes. In empirical implementations, these factors would be incorporated in an error term. In an early paper, Conlisk (1974) notes that the reduction of the variance of such an error term also can be thought of as equalization of opportunity. More importantly, such equalization may, in fact, increase the correlation between parental and child income or education and reduce intergenerational mobility.

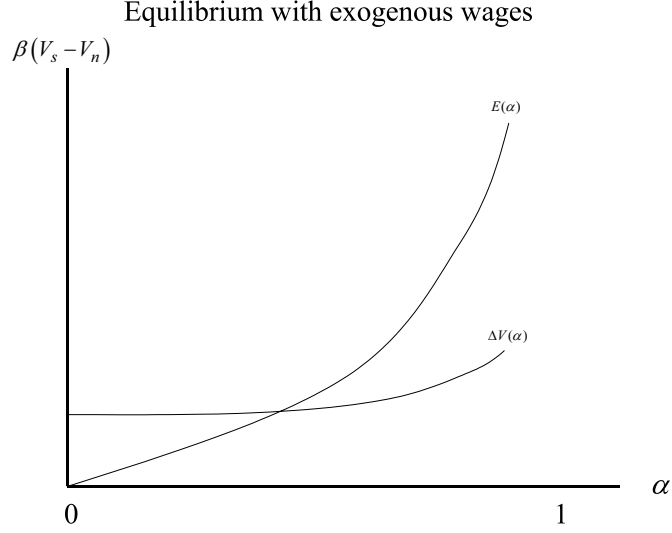


Figure 1:

tion of the  $E(\alpha)$  and the  $\Delta V(\alpha)$  curves described in Figure 1, satisfying

$$-\ln(1 - \alpha) = \beta \left( \ln I - \left( d_s - \frac{d_n}{I} \right) (\ln(1 - \alpha) + \alpha) \right). \quad (10)$$

Clearly, an increase in wage inequality  $I$ , shifts the  $\Delta V(\alpha)$  curve upwards, i.e., the additional value of being skilled rather than non-skilled increases for every value of  $\alpha$ . The intersection with the  $E(\alpha)$  curve therefore occurs at a higher level of  $\alpha$ . The relation between investment shares and inequality, defined by the above equation, is denoted  $\alpha = IN(I)$  and we call it the *equilibrium investment curve*. It is straightforward to show that  $IN(I)$  is monotonically increasing but bounded below unity; as inequality increases, the maximum share of income spent on education increases. At  $I = 1$ ,  $\alpha = 0$  and  $\frac{dIN(I)}{dI} = \beta$  while as  $I$  goes to infinity  $\alpha$  approaches unity.<sup>6</sup>

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<sup>6</sup>The LHS of (12) starts at zero with a slope  $\frac{1}{1-\alpha}$ , while the RHS starts at  $\beta \ln I > 0$  with a slope  $(d_s - d_n/I) \frac{\alpha}{1-\alpha} < \frac{1}{1-\alpha}$ . As  $\alpha$  approaches unity, the ratio of the LHS and the RHS is  $\frac{1}{\beta(d_s - \frac{d_n}{I})} > 1$ .

We can now conclude that inequality has two effects on upward mobility  $(1 - e_n) = \alpha d_n / I$ . There is a direct negative effect because the ability of unskilled parents to supply education to their children does not only depend on their own wage, but also on the wage of skilled workers. This is due to our assumption that educational services are produced by skilled workers and hence, education costs are indexed by  $w_s$ . We call this negative relation between inequality and upward mobility the *distance effect*. The other effect works through the response of  $\alpha$ . As inequality increases, the value of becoming educated increases, so  $\alpha$  increases. This raises upward mobility and we call it the *incentive effect*. This effect is due to the fact that parents care about the future welfare of their offspring and its strength therefore increases in the intergenerational discount factor. When inequality is sufficiently low, the incentive effect dominates. For example, at  $I = 1$ , we have  $\alpha = 0$  and as a result, the effect of inequality on upward mobility is given by:  $\frac{d(1-e_n)}{dI}|_{I=1} = d_n \frac{d\alpha}{dI}|_{I=1} = d_n \beta$ . However, due to the fact that  $\alpha$  is bounded below unity, the distance effect will eventually dominate.

In contrast to upward mobility, downward mobility and inequality of opportunity are monotonically related to inequality. Downward mobility decreases in inequality, while inequality of opportunity increases. Inequality has a direct positive effect on inequality of opportunity. In addition, the induced increase in  $\alpha$  further exacerbates the increase.

### 3.2 Endogenous wages

Let us now endogenize wages. When wages and utility differences are no longer constant, the above derivation of the maximum share of income invested in education is analytically cumbersome. In order to keep the analysis tractable, we therefore limit our discussion to the steady state of the economy. In the steady state, the above calculation of the relations between inequality, welfare and maximum investment shares, which are summarized by the



function  $\alpha = IN(I)$ , remains valid. However, it should be kept in mind that comparative statics analysis below is done across different steady states. Therefore, it is only suitable for cross-sectional analysis.

Wages will be determined on the competitive labor market. For simplicity, we assume that skilled workers can be used with a linear technology implying that  $w_s$  is independent of the supply of skilled. Non-skilled workers, on the other hand, have to work with a decreasing returns-to-scale technology. Therefore, their wage depends on the supply of non-skilled;<sup>7</sup>

$$w_n = a_n N^{-\sigma}, \quad (11)$$

with  $0 < \sigma < 1$  and inequality is given by

$$I = aN^\sigma, \quad (12)$$

where  $a \equiv \frac{w_s}{a_n}^{1/\sigma}$  is assumed to be larger than unity to ensure interior equilibria. By inverting this relation, we can express the ratio of skilled to non-skilled as a downward sloping convex function of inequality;

$$\frac{1 - N}{N} = S(I; a) \equiv aI^{-\frac{1}{\sigma}} - 1. \quad (13)$$

In a steady state, the downward and upward intergenerational flows must balance, which leads to the following steady-state condition

$$\frac{1 - e_n}{e_s} = S(I; a). \quad (14)$$

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<sup>7</sup>For simplicity, profits arising in this sector are assumed to be distributed to individuals in proportion to their income. This can be motivated by ownership of a fixed factor, complementary to non-skilled labor (land), being proportional to income. It is straightforward to show that the distribution of profits in this way has no consequence for the analysis and is therefore discarded.

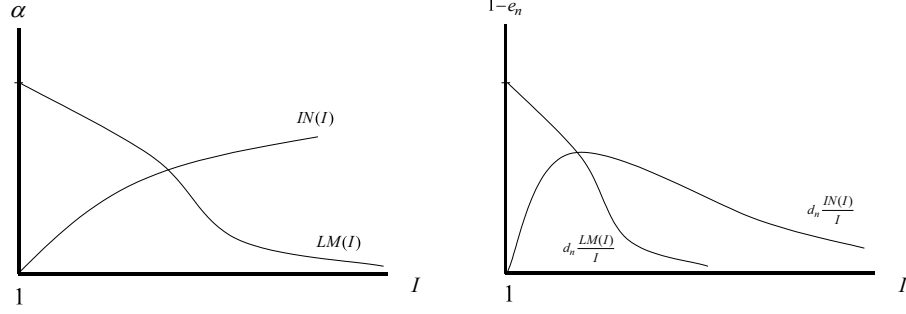


Figure 2:

Substituting (5) and (6) in the steady state condition, we derive a second equilibrium relationship between the maximum investment share  $\alpha$  and wage inequality  $I$ :

$$\alpha = \frac{IS(I; a)}{d_n + d_s IS(I; a)} \equiv LM(I). \quad (15)$$

The equation  $\alpha = LM(I)$  provides a negative relation between  $\alpha$  and  $I$ , which we call the *labor market equilibrium curve*. The reason why this curve is downward sloping can be explained in the following way. An increase in  $S$ , decreases inequality since the marginal productivity of unskilled increases. The direct effect of this on  $\frac{1-e_n}{e_s}$  is positive since  $(1 - e_n)$  depends positively on  $I$  due to the distance effect. However, the direct effect is not sufficient to restore equality and  $\alpha$  therefore has to increase.

The curves  $\alpha = IN(I)$  and  $\alpha = LM(I)$  have a unique intersection point, which determines the equilibrium under endogenous wages, as depicted in the left-hand panel of Figure 2. The same equilibrium is described in the right panel of the figure, but in terms of upward mobility, i.e., using the fact that  $1 - e_n = \alpha d_n / I$ . From the labor market condition,  $\alpha = LM(I)$ , we get

$$1 - e_n = \frac{S(I; a)}{1 + \frac{d_s}{d_n} IS(I; a)}, \quad (16)$$

which describes a negative relation between upward mobility and inequality. The other curve is derived from parents' decisions on education and is  $(1 - e_n) = IN(I)d_n/I$ . This is a hump shaped curve, reflecting the dominant incentive effect for low inequality and the dominant distance effect for high inequality, as discussed in the previous subsection.

## 4 Skill Biased Technical Change

Skill biased technical change and its effect have been at the center of recent discussions on wage inequality and skill distribution. In this section, we use our model to analyze the equilibrium responses to exogenous changes in the relative productivity of skilled.

Consider an increase in  $a$ , representing skill-biased technical change. From the definition of the  $\alpha = LM(I)$  curve, it follows that such an increase shifts it upwards, while having no effect on the curve  $\alpha = IN(I)$ . Hence, inequality and the share of income spent on education always increase. However, since the investment curve is bounded below unity, increases in the relative productivity of the skilled must eventually lead to smaller and smaller increases in the investment share. This is shown in the upper left panel of figure 3. In the upper right panel, we see that upward mobility increases when  $1 - e_n = d_n IN(I)/I$  is upward sloping, i.e., when inequality is low enough to make the incentive effect dominate. Eventually, however, the distance effect must dominate and then further increases in the relative productivity of the skilled lead to reductions in upward mobility. Hence, in a relatively equal economy, skill biased technical change increases upward mobility, while in a relatively unequal society, it is reduced.

The effect of skill biased technical change on skill distribution, namely on the ratio of skilled to unskilled workers, is ambiguous, since it rises with  $a$  and falls with  $I$ . It can be shown that as long as upward mobility rises,  $S$  increases as well. At high levels of inequality,

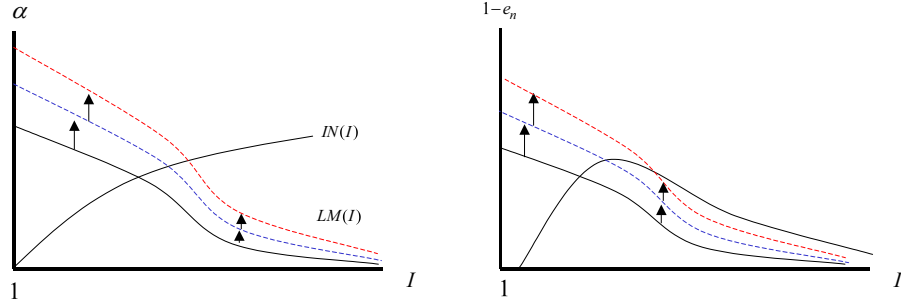


Figure 3: Relative productivity of the skilled increases

however, it is possible that  $S$  is reduced, due to the distance effect. Despite the fact that demand for skilled labor increases, the relative amount of skilled workers falls, since access to education for children of unskilled is sufficiently reduced. This result can shed light on some recent findings on reductions in relative supplies of skill together with the increase in wage inequality, as in Card and Lemieux (1999).

We can also evaluate how the equilibrium difference in individual welfare responds to the change in  $a$ . From (4), we know that the equilibrium difference in welfare is positively related to the investment share. Thus, an increase in  $a$  increases the difference in welfare between the skilled and the non-skilled. Note that this is due to two factors. First, wage inequality increases. Second,  $e_n - e_s = \alpha (d_s - d_n/I)$  increases, i.e., inequality of opportunity also increases. In other words, the importance of social background for welfare and the probability of becoming skilled increases.

## 5 Cultural and Educational Changes

In this section, we turn to the effects of changes in the productivity of education, which is measured by  $d_s$  and  $d_n$ . We first examine the following change in the economy: both  $d_s$  and  $d_n$  rise by the same proportion, so that their ratio remains unchanged. Next, we

consider the effect of an increase in  $d_n$  only, namely what happens if the cultural barrier to education faced by unskilled parents is reduced.

### 5.1 Better Education to All

A proportional increase in  $d_s$  and  $d_n$  means that the investment in education of both skilled and unskilled parents becomes more effective, but the relative effectiveness remains unchanged. Such a change leaves the curve  $E(\alpha)$  unchanged, while there is an upward shift in the curve  $\Delta V(\alpha)$ . Hence, holding wages fixed, the maximum share of educational spending  $\alpha$  increases, i.e., the curve  $\alpha = IN(I)$  shifts upwards. Thus,  $1 - e_s$  and  $1 - e_n$  rise by the same proportion and inequality of opportunity increases.

If wages are endogenous, greater investments in education and greater productivity of education lead to lower inequality, as can be seen in Figure 2. As noted, the  $\alpha = IN(I)$  curve shifts upwards, while the  $\alpha = LM(I)$  curve shifts downwards. Hence, inequality  $I$  declines, while the effect on the share  $\alpha$  is ambiguous. The long-run effect on upward mobility is, however, clear. The curve  $1 - e_n = d_n IN(I)/I$  shifts up, while the curve  $1 - e_n = d_n LM(I)/I$  remains unchanged. Hence inequality falls, while upward mobility increases. We therefore conclude that general changes in the productivity of education lead to a negative correlation between inequality and mobility.

### 5.2 Smaller Cultural Barriers to Education

Consider now an increase in  $d_n$  only. Namely, the productivity of supplying education rises for unskilled parents, while it remains unchanged for skilled parents. We first analyze the effect of this change with exogenous wages. The rise in  $d_n$  shifts the  $\Delta V(\alpha)$  curve downwards, reducing  $\alpha$ . As a result,  $e_s$  and downward mobility increase. The reason for these changes is that as  $d_n$  increases, the relative value of becoming skilled is reduced, since

it is no longer so bad to be unskilled, as the child of an unskilled can more easily move up the social ladder. The resulting negative incentive effect reduces  $\alpha$  and increases  $e_s$ . The reduction in  $\alpha$  also increases  $e_n$ , but there is a distance effect in the opposite direction, since the ability of non-skilled parents to provide education to their children increases with  $d_n$ . Using equation (10), we find that despite the decline in  $\alpha$ ,  $d_n\alpha$  still increases with  $d_n$ , i.e., the distance effect dominates.<sup>8</sup> Hence, when wages are exogenous, a reduction in the barrier to education for children of unskilled parents increases upward mobility. Clearly, inequality of opportunity falls.

When wages are endogenous, the results are more ambiguous, as we have to account for changes in inequality as well. As noted, the increase in  $d_n$  shifts the  $\alpha = IN(I)$  curve downwards, while  $1 - e_n = d_n IN(I)/I$  shifts upwards. As for the labor market curves, it is straightforward to see that the  $\alpha = LM(I)$  curve shifts down and to the left, while  $1 - e_n = d_n LM(I)/I$  shifts up and to the right. The intuition behind these shifts is as follows. When  $\alpha$  is held fixed and  $d_n$  increases there is more upward mobility. As a result, there are more skilled and wage inequality falls. Hence, the  $LM$  curve shifts to the left. If, on the contrary,  $1 - e_n$  is kept unchanged and  $d_n$  increases, that is possible only if the maximum spending on education,  $\alpha$ , declines. This means that downward mobility increases. As a result, there are less skilled workers and wage inequality rises. Hence, the upward mobility curve shifts to the right.

The downward shifts in the  $\alpha = IN(I)$  and  $\alpha = LM(I)$  curves imply that  $\alpha$  must fall. The effect on inequality  $I$  is not clear from this figure. However, we can show that unless  $\alpha$  is too large to begin with, inequality must decrease as  $d_n$  increases.<sup>9</sup> In that case, it is

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<sup>8</sup>The elasticity  $\frac{d\alpha}{d d_n} \frac{d_n}{\alpha}$  is given by  $\frac{d_n}{I} \frac{1}{\frac{1}{\beta\alpha} - (d_s - \frac{d_n}{I})} \frac{1-\alpha}{\alpha} \frac{\ln(1-\alpha)+\alpha}{\alpha}$  where  $\frac{d_n}{I} < 1$ ,  $\frac{1}{\frac{1}{\beta\alpha} - (d_s - \frac{d_n}{I})} < 1$  and  $\frac{1-\alpha}{\alpha} \frac{\ln(1-\alpha)+\alpha}{\alpha} \in [-\frac{1}{2}, 0]$ . Since the elasticity is larger than -1,  $\alpha d_n$  increases in  $d_n$ .

<sup>9</sup>In appendix 10.1, we show that inequality necessarily falls in  $d_n$  as long as  $\alpha < 0.68$ .

clear that an increase in  $d_n$  increases upward mobility,  $1 - e_n$ .

We have found that an improvement in the productivity of education for children of unskilled parents leads to a negative correlation between inequality and upward mobility: inequality falls and upward mobility rises. The effect on inequality of opportunity in this case is clear-cut – it falls as a result of lower cultural barriers to education for children of unskilled parents.

We conclude the analysis in this section by noting that changes in the productivity of education, or in the educational system, whether they favor all or only the unskilled, all lead to a negative correlation between inequality and mobility. This once more takes us to the empirical findings of Checchi, Ichino and Rustichini (1999), who find that the U.S. is less equal but more mobile than Italy. According to our results, this cannot be due to differences in education between the US and Italy, as the authors claim. According to our model, the difference can be attributed to a technological difference in  $a$ , namely in the relative productivity of skilled and unskilled. Another possibility that might be considered is that Italy has significant wage compression, due to strong unions, which reduces inequality  $I$ , even without changing productivity. We could interpret a lower value of  $a$  in Italy as reflecting higher wage compression. This can explain not only why Italy is more equal than the US, but also why it is less mobile, since lower inequality reduces mobility through the incentive effect.

This section describes changes in education without introducing public support for education. But most of the international differences in educational systems are due to different degrees of public support to education. Thus for example, large parts of education in the US are still privately financed, while public support for education in Europe is more widespread. Next, we wish to see how such differences in education affect inequality and

mobility. The next section therefore examines the effects of subsidization of education on inequality and mobility.

## 6 Educational Policy

In this section, we examine the effects of educational subsidies. We focus on policies aiming at increasing the share of children who become skilled by subsidizing education costs. Unlike, for example, Glomm and Ravikumar (1992), we do not assume that public education rules out private spending on education. Instead, we assume that parents are free to add to what the government provides. A key element in the analysis will, therefore, be the endogeneity of private expenditures.

In principle, we may distinguish between subsidies in kind, i.e., provision of public schools, and subsidies reducing the cost of education, for example tuition reductions or student loans. In our stylized model, however, such a distinction is not very meaningful since in both cases, we would effectively model such policy as changes in the educational productivity parameters  $d_j$ , as executed in the previous section. In contrast to the analysis in the previous section, however, we now take into account that increases in subsidies to education must be financed. All along, we assume financing to be achieved with a proportional income tax.

We now consider the case when public education is a perfect substitute to private education. Specifically, we assume educational costs to be as in the basic model, but are partly subsidized by the government.<sup>10</sup> The size of the subsidy is  $\pi w_s$  for every student, and financed by a proportional tax of rate  $T$ . Hence, the education thresholds are

$$e_j = 1 - \frac{\alpha w_j(1 - T) + \pi w_s}{w_s} d_j. \quad (17)$$

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<sup>10</sup>The alternative, when public subsidies are complements to private education, is analyzed in appendix.



The  $E(\alpha)$  curve, relating the threshold  $\alpha$  to the utility difference, remains unchanged, i.e.,  $\beta(V_s - V_n) = -\ln(1 - \alpha)$ , since a proportional tax has the same additive impact on utility for skilled and non-skilled.

Solving for the expected utility levels, we find that

$$\Delta V(\alpha) = \beta \left( \ln I - \left( d_s - \frac{d_n}{I} \right) (1 - T) (\ln(1 - \alpha) + \alpha) - (d_s - d_n) \pi \ln(1 - \alpha) \right).$$

Clearly,  $\Delta V(\alpha)$  rotates clockwise, shifting downwards for  $\alpha > 0$ , as  $T$  increases, since taxes reduce inequality of opportunity. On the other hand,  $\Delta V(\alpha)$  increases in  $\pi$  if  $d_s > d_n$ , since skilled parents are in a better position to take advantage of the public subsidy.<sup>11</sup> To find the combined effect, we use the steady state government budget constraint. This can be written  $T(w_s(1 - N) + w_n N) = \pi w_s(1 - N)$  implying  $\pi = T(1 + IS(I))/IS(I)$ .<sup>12</sup>

For low levels of inequality and provided  $d_s > d_n$ , the positive direct effect dominates and public subsidies shift  $\Delta V(\alpha)$  upwards. In that range, the  $\alpha = IN(I)$  curve shifts up, i.e., the incentive for education is *strengthened*. For higher levels of inequality, however, the effect through taxes become stronger and the sum of the effects is then of ambiguous sign. Clearly, if  $d_s - d_n$  is small, the negative tax effect is more likely to dominate and  $\alpha = IN(I)$  curve then shifts down.

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<sup>11</sup>It should be noted that the positive effect on  $\Delta V$  is larger in the case when subsidies complement private spending as, for example, when subsidies are given in the form of proportional reductions in tuition. See appendix.

<sup>12</sup>For convenience, we assume the government is unable to observe individual aptitude, therefore spending  $\pi w_s$  on all children who become skilled. Note also that we used the endogenous wage equilibrium condition  $(1 - N)/N = S(I)$  to derive the relation between  $\pi$  and  $T$ . This implies that in contrast to the case in section 3.1 the  $IN(I)$  curve can no longer be interpreted as a function of an exogenous wage inequality.

Next, we consider upward mobility,  $1 - e_n$ . The condition  $\Delta V(\alpha) = E(\alpha)$  can be written

$$\begin{aligned} & \ln \left( 1 - \left( \frac{1 - e_n}{d_n} - \pi \right) \frac{I(1 + IS(I))}{1 + IS(I)(1 - \pi)} \right) \\ &= \frac{\beta \ln I - \beta(d_s I - d_n) \left( \frac{1 - e_n}{d_n} - \pi \right)}{1 - \beta(d_s - d_n)\pi - \beta \left( d_s - \frac{d_n}{I} \right) \frac{1 + IS(I)(1 - \pi)}{1 + IS(I)}}, \end{aligned}$$

defining a curve relating upward mobility and inequality. It can be shown that when the subsidy increases, this curve shifts upward. The interpretation of this is that holding inequality constant, educational subsidies will crowd out private expenditures less than one-for-one.

The second condition is derived from the labor market equilibrium, equating the flows of upward and downward mobility. It is straightforward to show that this condition can be written

$$1 - e_n = \frac{S(I) + d_s S(I)(I - 1)\pi}{1 + \frac{d_s}{d_n} IS(I)},$$

defining a negative relation between upward mobility and inequality that shifts upward in  $\pi$ . When inequality low, ( $I$  close to unity), this labor market equilibrium curve becomes independent of  $\pi$ . Thus, if inequality is relatively low, inequality will fall with increases in the subsidy. We therefore conclude that changes in the degree of subsidization to the educational system lead to a negative correlation between inequality and mobility, at least in a relatively equal economy.

Considering, finally, inequality of opportunity, given by

$$e_n - e_s = \alpha(1 - T)(d_s - d_n/I) + \pi(d_s - d_n), \quad (18)$$

we see that it is affected by educational subsidies through several channels. First, there is the direct effect, which increases inequality of opportunity if  $d_s > d_n$ . Second, financing subsidies through proportional taxes reduces inequality of opportunity by weakening

the distance effect. Third, educational subsidies change the incentive for education and therefore the share of income parents are willing to spend on education. In general, the effect on  $\alpha$  is ambiguous but for low levels of inequality and if the advantage of skilled parents  $d_s - d_n$  is large,  $\alpha$  may increase leading to larger inequality of opportunity. Finally, wage inequality is affected by educational policies and to the extent that wage inequality is reduced, inequality of opportunity is reduced through the distance effect.

## 7 Redistributive taxation

Suppose now that the government uses income taxes to finance a fixed subsidy of size  $\tau w_s$  to each household. This may be interpreted as a redistributive transfer system or subsidies to basic education that everyone participates in. Disposable income for a household of type  $j$  then becomes

$$w_s \left( \frac{w_j}{w_s} (1 - T) + \tau \right)$$

and the aptitude thresholds become

$$e_j = 1 - \alpha \left( \frac{w_j}{w_s} (1 - T) + \tau \right) d_j.$$

Again, we find that the curve  $E(\alpha)$  is unaffected by  $T$  and  $\tau$ . The utility difference for given wage inequality  $\Delta V(\alpha)$  is

$$= \beta \left( \ln I_d - (1 - T + \tau) \left( d_s - \frac{d_n}{I_d} \right) (\ln(1 - \alpha) + \alpha) \right), \quad (19)$$

where

$$I_d \equiv \frac{1 - T + \tau}{\frac{1 - T}{I} + \tau}$$

denotes disposable income inequality. Using the government budget constraint,  $T(Nw_n + (1 - N)w_s) = \tau w_s$ , we get

$$\tau = T \frac{1 + IS(I)}{I + IS(I)} < T.$$

This implies that  $(1 - T + \tau)$  falls, while  $\frac{1-T}{I} + \tau$  increases, and hence  $I_d$  falls in  $T$ . Thus,  $\Delta V(\alpha)$  shifts downwards as  $T$  is increased, implying that the equilibrium curve  $\alpha = IN(I)$  shifts downwards. This is due to the fact that the progressive nature of the tax-transfer system reduces the incentives to become educated. Clearly, given  $I$ , the utility difference and the inequality of opportunity

$$\alpha (1 - T + \tau) \left( d_s - \frac{d_n}{I_d} \right),$$

fall. This policy increases downward mobility, while its effect on upward mobility

$$1 - e_n = \alpha (1 - T + \tau) \frac{d_n}{I_d}$$

is ambiguous, since the incentive effect and the distance effect push in opposite directions. It should be noted that for any  $I$ , a sufficiently high level of redistribution must increase  $e_n$ , since increasing redistribution eventually leads to  $I_d = 1$  and then  $\alpha = 0$  and  $e_n = 1$ .

The  $LM(I)$  curve, which can be written as

$$\alpha = \frac{IS(I)}{d_n + d_s IS(I) - (d_s - d_n) \frac{TS(I)(I-1)}{1+S}},$$

shifts upwards and to the right with  $T$ , since  $I > 1$  and  $d_s > d_n$ , as illustrated in figure 4. The reason for the shift is that when  $d_s > d_n$  redistribution implies that for a given  $\alpha$ , money will be less efficiently spent on education. As a result, fewer individuals become skilled, increasing wage inequality  $I$ . Hence, the curve shifts to the right. From the shifts of the two curves, we can conclude that the tax-transfer system necessarily increases  $I$ .

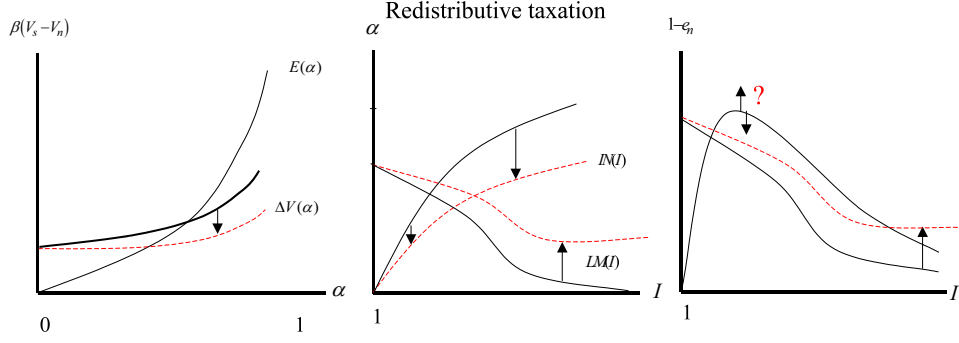


Figure 4:

Furthermore, if  $d_n$  is close to  $d_s$  and inequality is high, the shift in the  $\Delta V(\alpha)$  dominates so that  $\alpha$  falls, in which case inequality of opportunity must decrease.

The curve  $1 - e_n = LM(I)d_n \left( \frac{1-T}{I} + \tau \right)$  can be written as

$$1 - e_n = \frac{S(I)}{1 + \frac{d_s}{d_n} \frac{(1+S(I))IS(I) - T(I-1)S(I)}{1+S(I)+T(I-1)S(I)}},$$

which clearly shifts upward with  $T$ , while the effect on the curve  $1 - e_n = IN(I)d_n \left( \frac{1-T}{I} + \tau \right)$  is indeterminate. However, if inequality is low the latter curve shifts downward. The conclusion is that redistribution increases wage inequality, while its effect on upward mobility is ambiguous.

## 8 Unknown aptitudes

In the analysis above, we have assumed that educational aptitude is revealed before the educational decision is taken. In this section, we reverse this assumption and assume that educational decisions must be taken before aptitude is revealed. Most of the results discussed above turn out to be insensitive to this change in assumption. However, one qualitative difference arises. The resolution of the conflict of interest between parents and

children now depends on their relative bargaining strength, which therefore becomes a relevant variable in the model. Thus, we can study the consequences of cultural differences in family traditions across countries.

When aptitude is unknown, the child strictly prefers more spending to less since her chances of becoming skilled strictly increase in educational spending. The parent shares this interest with the child, but also cares about her own consumption. We assume that the level of educational spending is determined in an asymmetric Nash bargaining, where the threat of the parent is to pay nothing to the child, who then becomes non-skilled, while the threat of the child is to refuse to go to school. The bargaining power of the parent is parameterized by  $p \in [0, 1]$ .<sup>13</sup>

We assume that early in a child's education, it is revealed whether the investment is sufficient for the child to complete school. If not, the child can quit school, become non-skilled and investment cost is saved. Hence, parents' expected utility levels at the time of bargaining with their children are given by

$$\ln w_j + \beta V_n + (1 - e_j) (\ln (1 - \alpha_j) + \beta (V_s - V_n)), \quad (20)$$

where  $1 - e_j = \frac{\alpha_j w_j}{w_s} d_j$ , is the probability that a child with parents of type  $j$ , who invest  $\alpha_j w_j$  in education, becomes skilled. Children with parents of type  $j$  have expected utility levels of

$$V_n + (1 - e_j) (V_s - V_n).$$

Noting that the threat points are  $\ln w_j + \beta V_n$  and  $V_n$  for parents and children respectively,

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<sup>13</sup>When aptitude is known, parents pay the required amount for their childrens' education provided their is a non-negative surplus from doing so. Thus, the outcome is independent of individual bargaining strength.

the logarithm of the Nash-product is

$$\ln(1 - e_j) + p \ln[\ln(1 - \alpha_j) + \beta(V_s - V_n)] + (1 - p) \ln(V_s - V_n). \quad (21)$$

The first-order condition for maximizing (21) over  $\alpha_j$  is independent of  $j$  and hence, can be written in terms of a share  $\alpha$ , which is the same for skilled and unskilled parents:

$$\beta(V_s - V_n) = p \frac{\alpha}{1 - \alpha} - \ln(1 - \alpha) \equiv FOC(\alpha). \quad (22)$$

Clearly, (22) defines a unique investment ratio  $\alpha \in [0, 1]$ , which is an increasing function of the expected utility difference, but independent of wage and educational productivity. As above, there is a positive relation between educational investment and the future welfare difference between the skilled and the non-skilled. The strength of the incentive effect decreases with parental bargaining power.

Next, we calculate the utility difference between skilled and non-skilled individuals when educational investments are chosen optimally. Substituting (22) in equation (20) yields,

$$V_j = \ln w_n + \beta V_n + (1 - e_j) \frac{p}{\gamma} \frac{\alpha}{1 - \alpha},$$

implying

$$\beta(V_s - V_n) = \beta \left( \ln I + \left( d_s - \frac{d_n}{I} \right) p \frac{\alpha^2}{1 - \alpha} \right) \equiv \Delta V(\alpha). \quad (23)$$

Note that the utility difference between skilled and non-skilled individuals depends not only on wage inequality, but on the inequality of opportunity as well, since parents care about their children. Together, (22) and (23) define a unique investment share function of wage inequality. This function is increasing, just as the  $IN(I)$  function in the main model. It also leads to similar incentive and distance effects.

Combining the  $IN$  function in this case with the  $LM$  function, which is the same as in the main model, we derive equilibrium in a similar way. The main difference is

that it enables us to analyze the consequences of differences in parental bargaining power. This variable reflects family traditions and values in the society, which may differ across countries. Consider a country with a higher value of  $p$ . It can be shown that an increase in  $p$  reduces  $\alpha$  for any level of inequality. The intuitive reason for this result is that parents have more power and can afford more consumption, hence investing less in the education of their children. Hence, downward mobility is increased, upward mobility is reduced and inequality of opportunity falls as  $p$  increases.

Endogenizing wages, by requiring  $\alpha = LM(I)$  (which is independent of  $p$ ), we find that net downward mobility created by the increase in  $p$  increases the number of non-skilled workers and reduces the number of skilled workers. By drawing a figure similar to figure 2, this is straightforward since the relation between  $\alpha$  and  $I$  (the equivalent to the  $\alpha = IN(I)$  curve) shifts downwards, while the  $LM(I)$  curve remains unchanged. Hence,  $I$  rises as  $\alpha$  and upward mobility ( $1 - e_n$ ) fall. In this case, like in most cases of changes in the educational system analyzed above, we also get a negative correlation between inequality and upward mobility.

## 9 Conclusion

In this paper, we construct a model where wage and welfare inequality, intergenerational mobility and skill distribution are all endogenously determined in a general equilibrium framework. The model leads us to a number of important insights that may help understanding both empirical regularities and policy implications. One insight is that wage inequality has two effects on upward mobility, the incentive effect and the distance effect. The distance effect and its interaction with the incentive effect play an important role in this paper. While the incentive effect is straightforward, the distance effect needs some



discussion. We have assumed educational costs to be indexed to the wages of the skilled, motivated by the fact that labor input in education (teachers) largely consists of skilled workers. Hence, even if the wages of unskilled remain unchanged as the wages of the skilled rise, the distance effect would imply a negative effect on upward mobility. If, on the other hand, educational costs were indexed to non-skilled wages, the increase in skilled wages would reduce downward mobility while leaving upward mobility constant. Thus, in both cases, the distance effect causes a positive relation between wage inequality and inequality of opportunity.

Our model provides an explanation why countries differ with respect to inequality and mobility. We show that differences in the degree to which technology is skill-biased lead to positive correlation between inequality and mobility, while differences in education systems tend to lead to negative correlation between inequality and mobility. Thus, the empirical finding of Checchi, Ichino and Rustichini (1999), that US is more unequal but also more upward mobile than Italy, cannot be attributed to differences in educational systems, as they claim. Instead our model implies that it is either an indication that the US is more technologically skill-biased than Italy, or that Italy has less competitive labor markets with significant wage compression. The findings of Bjorklund, Eriksson, Jantti, Raaum and Osterbacka (2001), that the Nordic countries are more socially mobile than the US, in addition to being more equal, can be interpreted as an indication that public education is more extensive in these countries than in the US.

Another interesting result of our model is that general educational subsidies are likely to be effective in increasing the supply of skilled individuals in the economy, thereby reducing skill premia and wage inequality. If, however, an aim of such a policy is to reduce inequality of opportunity and make social background less important for lifetime outcomes, it is likely

to be less effective. If educated parents have a better ability to make the best use of the educational subsidy, children of educated parents benefit more than other children from public support to education. The increase in inequality of opportunity may increase welfare differences between children from skilled and non-skilled homes also if the policy reduces wage inequality.

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## 10 Appendix

### 10.1 Proof $d_n$ reduces inequality

In order to examine the effect on  $I$ , we use the labor market equilibrium (15), expressed as

$\frac{d_n}{I} = \frac{(1-d_s\alpha)S(I)}{\alpha}$ , + in equation (10) and get

$$\begin{aligned} \ln I &= -\frac{1}{\beta} \ln(1-\alpha) + \left( d_s - S(I) \left( \frac{1}{\alpha} - d_s \right) \right) (\ln(1-\alpha) + \alpha), \\ &= -\ln(1-\alpha) \left( \frac{1}{\beta} - d_s - d_s S(I) + \frac{S(I)}{\alpha} \right) + (d_s + S(I)d_s)\alpha - S(I). \end{aligned}$$

Now, we see that the derivative of the RHS with respect to  $I$  is

$$-S'(I) \left( \frac{1}{\alpha} - d_s \right) (\ln(1 - \alpha) + \alpha) \leq 0.$$

Thus, as long as the RHS increases in  $\alpha$ ,  $\frac{d\alpha}{dI} > 0$ , and since we already know that  $\frac{d\alpha}{dd_n} < 0$ , we conclude that an increase in  $d_n$  reduces inequality.

Now,

$$\begin{aligned} \frac{\partial RHS}{\partial \alpha} &= \frac{1}{1 - \alpha} \left( \frac{1}{\beta} - d_s - d_s S(I) + \frac{S(I)}{\alpha} \right) \\ &\quad + \ln(1 - \alpha) \frac{S(I)}{a^2} + d_s + S(I) d_s, \\ &= \frac{1}{1 - \alpha} \left( \frac{1}{\beta} - d_s \right) + \frac{S(I)}{\alpha} \left( \frac{\ln(1 - \alpha)}{\alpha} + \frac{1}{1 - \alpha} - \frac{d_s \alpha^2}{1 - \alpha} \right) + d_s. \end{aligned}$$

This is positive for sufficiently low  $\alpha$  since

$$\lim_{\alpha \rightarrow 0} \frac{S(I)}{\alpha} \left( \frac{\ln(1 - \alpha)}{\alpha} + \frac{1}{1 - \alpha} - \frac{\alpha^2 d_s}{1 - \alpha} \right) = \frac{S(I)}{2} > 0,$$

and the other terms are always positive. Furthermore, the term

$$\frac{\ln(1 - \alpha)}{\alpha} + \frac{1}{1 - \alpha} - \frac{d_s \alpha^2}{1 - \alpha} > \frac{\ln(1 - \alpha)}{\alpha} + \frac{1}{1 - \alpha} - \frac{\alpha^2}{1 - \alpha} > 0$$

for  $\alpha < 0$ , we can also find a bound on  $\alpha$ , ensuring that all terms remain positive.

$$\frac{\ln(1 - \alpha)}{\alpha} + \frac{1}{1 - \alpha} - \frac{d_s \alpha^2}{1 - \alpha} > \frac{\ln(1 - \alpha)}{\alpha} + \frac{1}{1 - \alpha} - \frac{\alpha^2}{1 - \alpha},$$

which is positive for  $\alpha$  smaller than 0.683.

## 10.2 Imperfect information

Consider now the intermediate case to full and no information. We assume that each family receives a signal on the aptitude of their child. The signal (with a value  $s$ ) is true with

probability  $\pi$  and false with probability  $1 - \pi$ ; if the signal has no informational value, i.e., it is uniformly distributed on  $[0, 1]$ . This assumption nests the previous models as special cases when  $\pi$  is zero or unity, respectively.

Consider now a parent of type  $j$  who receives a signal  $s$  and invests a share  $\alpha$  of its income on education. The child will succeed if  $\delta_j \alpha \geq 1 - e$ , where  $\delta_j \equiv d_j \frac{w_j}{w_s}$ . Let  $Q$  denote the probability of success. Then,  $Q(\alpha) = \pi + (1 - \pi) \delta_j \alpha$  if  $\delta_j \alpha \geq s$ , and  $Q(\alpha) = (1 - \pi) \delta_j \alpha$  if  $\delta_j \alpha < s$ .

Thus, assuming that the bargaining power of the parents is  $p$ , the Nash bargaining solution is the level of  $\alpha$  that maximizes

$$\ln \text{NASH}(\alpha) = \ln Q(\alpha) + p \ln((\ln(1 - \alpha) + \beta(V_s - V_n))) + (1 - p) \ln(V_s - V_n).$$

The complication here is that the function  $Q(\alpha)$  and therefore also the Nash product are discontinuous at  $\alpha = \frac{s}{\delta}$ . We thus decompose the Nash product into its two continuous parts;

$$\begin{aligned} M(\alpha) &= \ln(\pi + (1 - \pi) \delta \alpha) \\ &\quad + p \ln(\ln(1 - \alpha) + \beta(V_s - V_n)) + (1 - p) \ln(V_s - V_n), \\ L(\alpha) &= \ln((1 - \pi) \delta \alpha) \\ &\quad + p \ln(\ln(1 - \alpha) + \beta(V_s - V_n)) + (1 - p) \ln(V_s - V_n). \end{aligned}$$

$\ln \text{NASH}(\alpha) = M(\alpha)$ , if the effective investment ( $\delta \alpha$ ) is at least as large as the signal, and  $L$  is the one that applies if the effective investment is less than the signal. Note that  $L$  is the log Nash product in the complete uncertainty case, just scaled by  $(1 - \pi)$ .

Now, consider the function  $M$  and denote  $\alpha_M(\delta, \pi) = \arg \max_{\alpha \in [0, \bar{\alpha}]} M(\alpha)$ . If  $\pi \geq \frac{\delta \beta(V_s - V_n)}{p + \delta \beta(V_s - V_n)}$ ,  $M$  is decreasing in  $[0, \bar{\alpha}]$  and  $\alpha_M(\delta, \pi) = 0$ . Otherwise, it satisfies first order

condition, i.e.,

$$\ln(1 - \alpha_M) + \beta(V_s - V_n) = p \frac{\alpha_M}{1 - \alpha_M} \left( 1 + \frac{\pi}{(1 - \pi) \delta \alpha_M} \right).$$

Now let us look at  $L$ . Clearly,  $M(\alpha) \geq L(\alpha)$  for all  $\alpha$  in  $[0, \bar{\alpha}]$  with strict inequality if  $\pi > 0$ . Denote  $\alpha_L \equiv \arg \max L(\alpha)$ . Since  $L$  equals 21 we know that  $\alpha_L$  satisfies the first-order condition,

$$\ln(1 - \alpha_L) + \beta(V_s - V_n) = p \frac{\alpha_L}{1 - \alpha_L}.$$

Finally, define  $\hat{\alpha}(\delta, \pi)$  as the value of  $\alpha$  such that the value of  $M$  equals the maximum value of  $L$  and  $\hat{\alpha}$  larger than  $\alpha_L$ . (i.e.,  $M(\hat{\alpha}(\delta, \pi)) = L(\alpha_L)$ ). Note that  $\bar{\alpha} > \hat{\alpha}(\delta, \pi) > \alpha_L > \alpha_M(\delta, \pi) \geq 0$ . Furthermore, while  $\alpha_M(\delta, \pi)$  depends on  $\delta$  and  $\pi$ ,  $\alpha_L$  does not.

Now recall that given a value of the signal  $s$ , either  $\alpha\delta$  is larger than  $s$  and  $NASH(\alpha) \equiv M(\alpha)$ , or  $\alpha\delta$  is smaller than  $s$ , in which case  $NASH(\alpha) \equiv L(\alpha)$ . Thus, at  $\alpha = \frac{s}{\delta}$ , there is a discrete fall in  $NASH(\alpha)$ .

Consequently, we can state that the value of  $\alpha$  that maximizes  $NASH(\alpha)$  is

$$\arg \max_{\alpha} Nash(\alpha) = \begin{cases} \alpha_M(\delta, \pi) & \text{if } s < \delta \alpha_M(\delta, \pi) \\ \frac{s}{\delta} & \text{if } \delta \alpha_M(\delta, \pi) \leq s \leq \delta \hat{\alpha}(\delta, \pi) \\ \alpha_L & \text{if } s > \delta \hat{\alpha}(\delta, \pi) \end{cases}$$

In figure 5, we can see that if the signal is smaller than  $\delta \alpha_M(\delta, \pi)$  (indicating a very apt child), the maximum of the Nash product is attained at  $\alpha_M$ . For intermediate signals  $s \in [\delta \alpha_M, \delta \hat{\alpha}]$ ,  $M$  is decreasing, and the global maximum is attained at  $\frac{s}{\delta}$ . Note that for high values of  $\pi$  (i.e., when the signal is true with high probability) the function  $M$  is always decreasing, and the first region does not exist. Finally, if the signal is larger than  $\delta \hat{\alpha}$ , indicating a less apt child, the value of  $L$  evaluated at its maximum is larger than the value of  $M$  evaluated at  $\frac{s}{\delta}$ , in which case the investment share is  $\alpha_L$ .

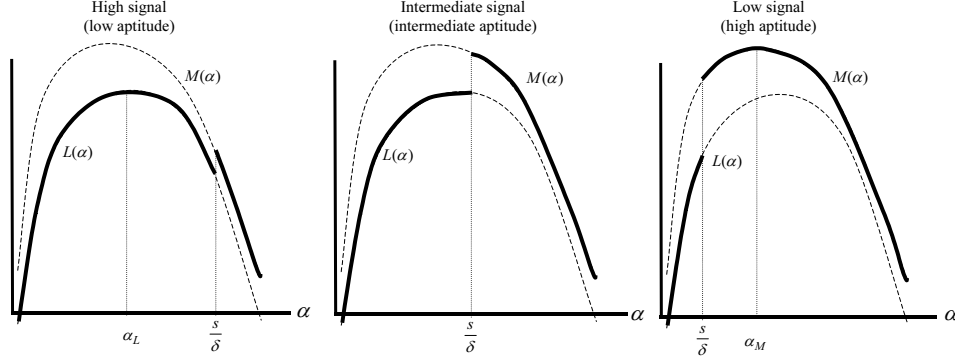


Figure 5:

We should note that the share of income devoted to education is a non-monotonic function of the signal. For low values of the signal (indicating a very apt child) the share of investment is independent of the signal and increasing in  $\delta$ . Thus, skilled parents spend a higher share of a higher income on their childrens' education. For higher signals, the income share spent on education increases in the signal and decreases in  $\delta$ ; parents invest as much as would be necessary for the child to pass school if the signal were true. If the signal becomes sufficiently high, the investment share is again independent of the signal. The parent puts all her hopes into the probability that the signal is false, investing the same level as she would have done without any information about her child's aptitude.

### 10.3 Complementary public support to education

In this section, we assume that public education is complementary to private education.<sup>14</sup>

More specifically, if the government spends  $\pi w_s$  on public education, a child with aptitude

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<sup>14</sup> A subsidy in the form of a price reduction on education, like a proportional tuition reduction or subsidized student loans, has effects qualitatively identical to those of complementary public education. Subsidies targeted at children from non-skilled homes have qualitatively similar effects to those of increases in  $d_n$ .

Details available upon request from the authors.



$e$  to a parent of type  $j$ , who pays a share  $\alpha$  of income for private education, can become skilled if

$$\pi(1-T)\alpha w_j \geq w_s \frac{1-e}{d_j}, \quad (24)$$

implying that the thresholds for education are  $e_j = 1 - \pi(1-T)\alpha w_j d_j / w_s$ . We can interpret  $\pi$  as the amount of educational services provided by the government and  $(1-T)\alpha w_j / w_s$  as the amount purchased by the marginal parents.

Again, we find that the curve  $E(\alpha)$ , as given by (4), remains unchanged and independent of  $\pi$ . On the other hand,  $\pi$  clearly affects inequality of opportunity, implying

$$\Delta V(\alpha) = \beta \left( \ln I - \pi(1-T) \left( d_s - \frac{d_n}{I} \right) (\ln(1-\alpha) + \alpha) \right),$$

which increases in  $\pi$  and falls in  $T$ .

We next wish to examine the effects of changes in subsidy  $\pi$  on the parents' educational decisions. It is clear that this effect depends on how  $\pi(1-T)$  changes with the subsidy, namely on the budget constraint of the government. This is given by  $T(w_n N + w_s(1-N)) = \pi w_s(1-N)$ . Dividing by  $N$  and  $w_n$  and using the relation  $\frac{1-N}{N} = S(I)$ , we have  $T = \frac{\pi IS(I)}{1+IS(I)}$ , implying

$$\pi(1-T) = \pi \frac{1+IS(I)(1-\pi)}{1+IS(I)},$$

which increases whenever  $1+IS(I)(1-2\pi) > 0$ .

Thus,  $\pi(1-T)$  first increases in  $\pi$  but will eventually fall. Below, we focus on the case when  $\pi$  is low enough to imply that  $\pi(1-T)$  increases in  $\pi$ . Then, increases in  $\pi$  shift the  $\Delta V(\alpha)$  curve upwards, increasing inequality of opportunity and the utility difference between the skilled and non-skilled for each  $\alpha$ . Thus, equilibrium  $\alpha$  increases for each level of inequality. In other words, the  $IN(I)$  curve shifts upwards.

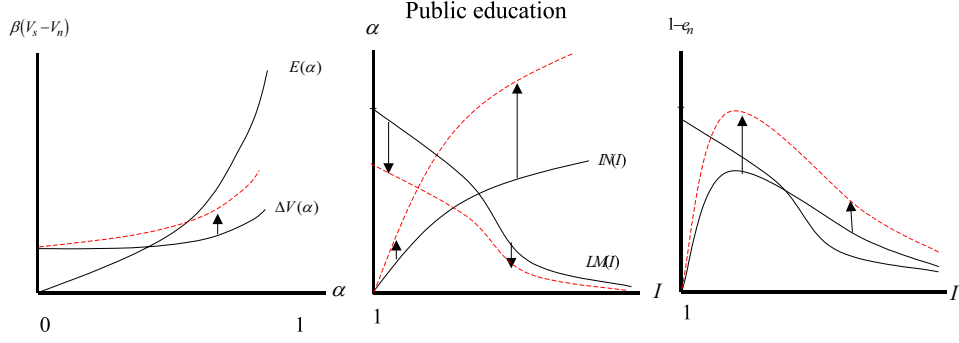


Figure 6:

The labor market equilibrium curve  $LM(I)$  with public education is given by

$$\alpha = \frac{1}{\pi(1-T)} \frac{IS(I)}{d_n + d_s IS(I)},$$

which shift downwards as the subsidy to education is increased. The intuitive reason for this shift is the following. For a given level of  $\alpha$ , increasing the subsidy means greater spending on education by everyone and hence, upward mobility rises and downward mobility falls. As a result, wage inequality declines and the curve shifts to the left. However, labor market equilibrium in terms of upward mobility

$$1 - e_n = \alpha \frac{d_n \pi (1 - T)}{I} = \frac{S(I)}{1 + \frac{d_s}{d_n} IS(I)},$$

is unaffected by the subsidy.

The consequences of an increase in  $\pi$  at levels low enough to imply that also  $\pi(1-T)$  increases, are illustrated in figure 6. Unambiguously, wage inequality and upward mobility are negatively correlated, just as in all changes in Section 5. When the subsidy is small, inequality is reduced and upward mobility rises. For sufficiently high subsidies, the outcome is reversed, so that inequality is increased and upward mobility reduced.

As for inequality of opportunity, described by  $\pi(1-T)\alpha(d_s - \frac{d_n}{I})$ , we have two con-

flicting effects. The first is due to the fact that the subsidy is more effective for skilled parents, as they invest more in private education. This effect increases the inequality of opportunity. The second effect is that due to the subsidy, more individuals become skilled, thereby reducing wage inequality. This reduces the distance effect for the unskilled and thus also inequality of opportunity. Hence, the overall effect of subsidy on inequality of opportunity is ambiguous.

We should note that the “perverse” case, when educational subsidy increases inequality of opportunity, tends to be more likely when the difference  $d_s - d_n$  is large. In this case, the ability of the skilled to use subsidies to education is far greater than that of the non-skilled and thus, a larger subsidy leads to higher inequality of opportunity .

#### 10.4 Proportional Subsidy

Consider proportional public support to education, financed by an income tax at a rate  $T$ . The rate of the subsidy is  $g$  of private investment in education. Hence, private plus public educational investments in a child with parent of type  $j$  are  $(1 + g)(1 - T)w_j\alpha$  and hence, the threshold levels of aptitude are

$$e_j = 1 - d_j\alpha \frac{(1 + g)(1 - T)w_j}{w_s},$$

implying that the share of children of type  $j$  who become skilled is proportional to  $\alpha$ . From the previous equation, we see that the  $IN(I)$  curve shifts upwards in  $g$  if and only if  $(1 + g)(1 - T)$  increases in  $g$ . As in the case of public education, increases in  $g$  first increases and then decreases  $(1 + g)(1 - T)$  when  $I$  is kept constant. This implies that an increase in  $g$  from a sufficiently low level leads to an upward shift in the  $IN(I)$  curve. The  $LM(I)$  curve with subsidies shifts downwards, while labor market equilibrium in terms of  $e_n$  is unaffected by the subsidy. Thus, the qualitative effects of proportional subsidies are identical to those

of complementary public education.